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No one interested in recent efforts to bring precision and order into the conceptions of mathematics can fail to commend the purpose of the joint authors or to agree with them in their view of the unsatisfactory character of the usual definitions. On the other hand, the book fails, I think, to present any convincing and novel constructive results. The ideas of the authors are not presented either with the clearness or the acumen necessary to make them effective and are not as new as the authors suppose. For example, the main thesis of the authors regarding functional relation, to wit, that it often involves likeness of order of the corresponding quantities of the variables has been expressed with far greater exactness and logical completeness by Russell in his "Principles of Mathematics," Chapter XXXII (On the Correlation of Series); while, on the other hand, the argument (pages 151, 152) by means of which they seek to restrict the notions of variable and function to relations between *ordered* aggregates is not likely to convince "the Peano school" against whom it is directed. Philosophically speaking, a variable denotes *any* term standing in a given relation and a function denotes the relation in question. The definitions of Richardson and Landis simply restrict the broader definitions to more special cases. But the broader definitions should be insisted upon for the sake of the logical unification which they confer upon the various branches of mathematics. And when it comes to the point of giving a constructive definition of variable, the authors resign the effort on the plea that it would require a too lengthy discussion of the philosophical doctrine of the categories (page 154 et seq.).

The consideration of quantity yields an equally disappointing result. Of all the concepts of mathematics requiring clarification, this is the most flagrant; yet, so far as I can see, the present authors only add to the confusion.

More in accord with what the reviewer would regard as sound doctrine is the stand taken against the symbol theory of pure mathematics. For, even before any real objects satisfying the variables in a mathematical expression have been found, the expression is not a mere symbol, but indicates certain operations or relations between *possible* objects. Good remarks are also to be found concerning the distinction between independent and dependent variables (page 175).

Taken as a whole, what the book lacks is exactly what it aims to embody—a philosophical background for the criticism of mathematical ideas. One representative illustration of this is the naïve discussion of similarity and identity on pages 5 and 6. But after all, this is only the first and preliminary volume.

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The Arithmetical Philosophy of Nicomachus of Gerasa. By GEORGE JOHNSON, Ph.D. (University of Pennsylvania dissertation.) New Era Printing Company, Lancaster, Pa. 1916. 49 pages.

This pamphlet presents a partial translation (the complete translation is deposited in the library of the University of Pennsylvania) of the famous *Intro-*

duction to Arithmetic of Nicomachus, the earliest extant separate essay at a systematization of the Greek science of Arithmetikê, treating of numbers as such, as distinguished from Logistikê, which treated of numerical reckoning from a practical standpoint. Although the *Introduction* was no such landmark in the history of mathematical advance as the works of Euclid, or Archimedes, or Diophantus, or of some few other Greeks, it is still an extremely important document in educational mathematics and continued to exercise a great influence in the Latin versions of Apuleius and Boethius.

Dr. Johnson's dissertation is not a pretentious performance, and its aim seems to be description only, for he has not added to the translation, which covers a selection of passages designed to show the general argument of the *Introduction,* any independent treatment of Nicomachus himself, his philosophy, or his number theory. The commentary which he presents is confined to summaries and explanatory matter connecting the translated passages (for the footnote to the title is the only one in the book), and is largely based on Nesselmann and Cantor. Recognizing the generally descriptive nature of the commentary, the reader will still find statements that need modification. It does not give a true idea of the situation to say that the *Theologumena Arithmetica,* which as it stands is a patchwork of passages from several hands, is "probably the work of Nicomachus" (p. 1), although it certainly is largely Nicomachean; nor does it mean much to say that Nicomachus, long before algebra was developed, was "among the first to attempt a systematic treatment of arithmetic distinct from algebra" (p. 2). The statement that the number contrasted by Nicomachus with his "scientific" number is the "ideal number" of Plato (p. 6) surely calls for proof, for the divine numbers which are discussed in the *Theologumena Arithmetica* ought to be brought into consideration, as well as the Platonic. Some of the formulas, also, might be criticized. $(2n - 1)/n$, for example, hardly comprehends all possible superpartials (defined in the treatise as the relation between two numbers "whenever a number contains in itself the whole compared and in addition more of its parts than one," I. 20. 1), and it is a bad slip to give $a : \sqrt{ab} = \sqrt{ab} : b$ as the formula of the "arithmetic proportion." The author may be misled by the free use of the term "proportion" among the ancients; but the "arithmetic proportion" is at any rate identical with our "arithmetic progression," wherein $a - b = b - c$.

The translation is after all the raison d'être of the dissertation. It is uneven in character, the chief fault being that Dr. Johnson follows Hoche's text too literally and thus is led into needlessly anacoluthic English (*e. g.*, Book II, Chap. 1, Section 1, and II. 27. 1) and, in a few cases, into translating a hopelessly corrupt text (*e. g.*, I. 13. 8, where ἑαυτοῦ is certainly preferable to ἑαυτῶν; II. 23. 5, where Ast's text is far better than the hopeless reading of Hoche). Somewhat more care in punctuation and in proof-reading would have saved the sense of several ambiguous passages (see, *e. g.*, I. 9. 1; and on p. 42, lines 15 ff.). But beyond this the version is not free from errors, of which the citation of a few of the most serious must suffice here. In I. 7. 4 Nicomachus states that even numbers, when divided into two parts, show but "one of the two kinds" (τὸ ἕτερον εἶδος) of number in these parts; that is, both are odd or both even; Dr. Johnson says, "shows the first kind of number only (odd) without share in the rest." This is obviously false; because, for example, $8 = 5 + 3$ or $6 + 2$. Nicomachus tells in I. 12. 1–2 of the derivation of "composite numbers," using the word συντιθέναι in the sense of "to multiply" in the context; the translator gives the sense as "to add" and thereby creates confusion. Another serious error comes in the discussion of the "sieve of Eratosthenes" (I. 13). The Greek reads that the odd numbers "assume in turn the measuring function" (p. 33, 2, Hoche: ὥστε τὸ μὲν μετρεῖν διαδέξονται, κτλ.) in the process of discovering which terms are prime, but the translator has distorted the simple phrase into "their capacity of measuring will depend on the order in which they (*i. e.*, numbers to be measured) lie in the line" (p. 16). Still another slip gives the reader a fundamental misconception of Nicomachus's method of discovering "perfect numbers" (I. 16. 4). Dr. Johnson (p. 19) thus summarizes up the rule: "Sum up the powers of 2 and add unity to each sum; when the result is prime, multiply it by the last power of 2 in the series summed up; the product will be a perfect number." The translated section which follows is in conformity with this notion. But there is really no question of "adding unity to each sum"; εἶτα ἀεὶ κατὰ ἑνὸς πρόσθεσιν ἐπισωρεύειν (p. 41, 6, Hoche) means "then you must sum them up, as you take them one by one into the combination," as a comparison of the commentary by Iamblichus (*in Nic.*, p. 34, 2, edited by Pistelli) will amply confirm. The translator has probably failed to notice that unity is definitely included by Nicomachus in the series of (even-times even) numbers which he here uses, a fact which is made perfectly clear by the opening words of section 8 of the chapter. One leaves the reading of this dissertation with the feeling that the treatment accorded to Nicomachus is not as thorough-going nor as accurate as he deserves.

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ALGEBRA.

477. Proposed by J. L. RILEY, Northeastern State Normal School, Tahlequah, Oklahoma.

Evaluate the product $(1 + r + r^2 + r^3)(1 + r^2 + r^4 + r^6) \cdots (1 + r^{2^{n-1}} + r^{2^n} + r^{3\cdot 2^{n-1}})$.

478. Proposed by CLIFFORD N. MILLS, Brookings, S. Dakota.

Solve the equations

$$\frac{l}{x} + \frac{y}{m} + \frac{z}{n} = 1, \qquad \frac{x}{l} + \frac{m}{y} + \frac{z}{n} = 1, \qquad \frac{x}{l} + \frac{y}{m} + \frac{n}{z} = 1.$$